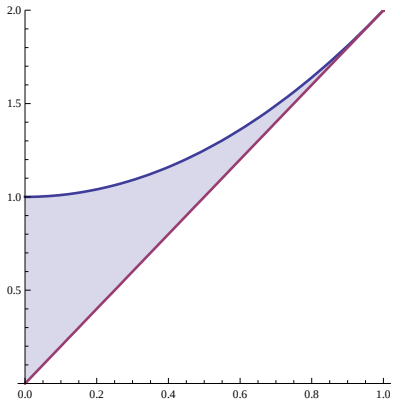
Math 142 **Test #2** **Spring 2010**

You must show all your work to receive full credit, especially when differentiating and integrating functions. If you are using a TI-89, you may use it to check answers, but you must show how you found derivatives and integrals.

1. (10 pts) Consider the planar region of density ρ bounded by the curves with equations $x = 0$, $y = 2x$, and $y = x^2 + 1$. See the diagram below.



For this region,

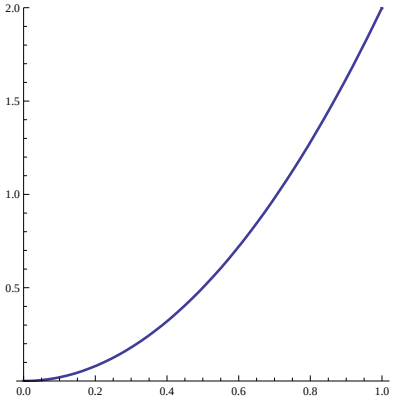
   (a) Find M_x.

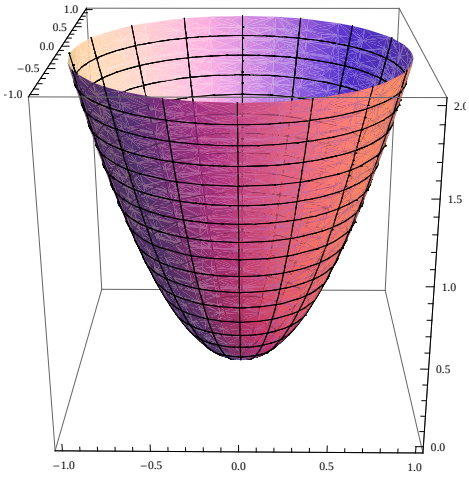
   (b) Find M_y.

   (c) Find m.

   (d) Find the centroid.

2. (10 pts) The shape of a bowl is the solid of revolution created by rotating the parabola $y = 2x^2$, $0 \le x \le 1$, about the y-axis, where x and y are measured in feet. See the diagrams below.





Find the amount of work done in filling the bowl with water that is initially at the level $y = 0$. Let ρ be the density of water.

3. (10 pts) Find the following indefinite integrals.

   (a) (10 pts) $\displaystyle\int x \sin 2x \, dx$

   (b) (10 pts) $\displaystyle\int \frac{1}{x^2 + 2x} \, dx$

   (c) (10 pts) $\displaystyle\int \frac{1}{\sqrt{x^2 - 4}} \, dx$

4. (10 pts) A population $P(t)$ increases at a rate proportional to $t - 5$, where t represents time, in months.

   (a) (10 pts) Write a differential equation that describes the growth of P.

   (b) (10 pts) Solve the differential equation in part (a) for the general solution.

   (c) (10 pts) Suppose that P is 15 when $t = 0$ and P is 20 when $t = 5$. Find the particular solution that satisfies these conditions.